

Proposition 6.2.6: For all sets  $A, B, C$ ,

if  $A \subseteq B$  and  $B \subseteq C^c$ , Then  $A \cap C = \emptyset$ .

Proof: Let  $A, B, C$  be any sets.

Suppose  $A \subseteq B$  and  $B \subseteq C^c$ ,  
[NTS:  $A \cap C = \emptyset$ ]

Suppose, by way of contradiction,  
that  $A \cap C \neq \emptyset$ .

- $\therefore$  There exists an element  $x \in U$  such that  $x \in (A \cap C)$ .
- $\therefore x \in A$  AND  $x \in C$  by definition of "Intersection".
- $\therefore x \in C$  by specialization.
- $\therefore x \in A$  by specialization.
- $\therefore$  Since  $A \subseteq B$ ,  $x \in B$  by UNIVERSAL Modus Ponens.
- $\therefore$  Since  $B \subseteq C^c$ ,  $x \in C^c$  by UNIVERSAL Modus Ponens.
- $\therefore x \notin C$  by definition of "Complement".
- $\therefore x \in C$  and  $x \notin C$  by Conjunction, and this is a contradiction.
- $\therefore A \cap C = \emptyset$  by proof-by-contradiction.

QED, By Direct Proof.

The "QED, by Direct Proof" is an abbreviation of the following:

$\therefore$  For all sets  $A, B, C$ , if  $A \subseteq B$  and  $B \subseteq C^c$ ,  
then  $A \cap C = \emptyset$ , by Direct Proof.  
QED